



Two temperature generalized magneto-thermoelastic interactions in an elastic medium under three theories



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ABSTRACT

Magneto-thermoelastic interactions in an isotropic homogeneous elastic half-space with two temperatures are studied using mathematical methods under the purview of the Lord–Shulman (LS) and Green–Lindsay (GL) theories, as well as the classical dynamical coupled theory (CD). The medium is considered to be permeated by a uniform magnetic field. The general solution obtained is applied to a specific problem of a half-space and the interaction with each other under the influence of magnetic field subjected to one types of heating the thermal shock type. The normal mode analysis is used to obtain the exact expressions for the displacement components, force stresses, temperature and couple stresses distribution. The variations of the considered variables through the horizontal distance are illustrated graphically. Comparisons are made with the results between the three theories. Numerical work is also performed for a suitable material with the aim of illustrating the results.

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1. Introduction

The classical coupled thermoelasticity theory proposed by Biot [1] with the introduction of the strain-rate term in the Fourier heat conduction equation leads to a parabolic-type heat conduction equation, called the diffusion equation. This theory predicts finite propagation speed for elastic waves but an infinite speed for thermal disturbance. This is physically unrealistic. To overcome such an absurdity, generalized thermoelasticity theories have been propounded by Lord and Shulman [2] as well as Green and Lindsay [3] advocating the existence of finite thermal wave speed in solids. These theories have been developed by introducing one or two relaxation times in the thermoelastic process, either by modifying Fourier's heat conduction equation or by correcting the energy equation and Neuman–Duhamel relation. According to these generalized theories, heat propagation can be visualized as a wave phenomenon rather than a diffusion one; in the literature, it is usually referred to as the second sound effect. These three theories are structurally different from one another and one can not be obtained as a particular case of the other. Various problems characterizing these theories have been investigated and has revealed some interesting phenomenon. Brief reviews of this topic have been reported by Chandrasekharaiah [4,5]. The coupled theory of thermoelasticity has been extended by including the thermal relaxanon time in the constitutive equations by Lord and Shulman [2] and Green and Lindsay [3]. These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermoelasticity. This exist in the following differences between the two theories

1. The Lord–Shulman theory (L–S) involves one relaxation time of thermoelastic process (τ_0). The Green and Lindsay (G–L) involves two relaxation times (τ_0, ν_0).

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Nomenclature

λ, μ	counterparts of Lamé's parameters
p	initial pressure
η	initial stress parameter
a	two temperature parameter
α_t	coefficient of linear thermal expansion
$\theta = T - T_0$	thermodynamical temperature
$\phi = \phi_0 - T$	conductive heat temperature (thermal temperature)
T	absolute temperature
T_0	temperature of the medium in its natural state assumed to be $ \frac{T-T_0}{T_0} < 1$
σ_{ij}	components of the stress tensor
u_i	components of the displacement vector
ρ	density of the medium
e_{ij}	components of the strain tensor
e	cubical dilatation
C_E	specific heat at constant strain
K	thermal conductivity
τ_0	thermal relaxation time
μ_0	magnetic permeability
ε_0	electric permittivity
F_i	Lorentz force
δ_{ij}	Kronecker delta function

2. The LS energy equation involves first and second time derivatives of strain, whereas the corresponding equation in GL theory needs only the first time derivative of strain.
3. In the linearised case according to the approach of (G–L) theory the heat cannot propagate with finite speed unless the stresses depend on the temperature velocity, whereas according to (L–S) theory the heat can propagate with finite speed even though the stresses there are independent of the temperature velocity.
4. The Lord–Shulman theory (L–S) can not be obtained from Green and Lindsay (G–L) theory.

The study of the electromagneto-thermoelastic interactions which deals with the interactions among the strain, temperature and the electromagnetic field in an elastic solid is of great practical importance due to its extensive uses in diverse field, such as geophysics (for understanding the effect of the Earth's magnetic field on seismic waves), damping of acoustic waves in a magnetic field, designing machine elements like heat exchangers, boiler tubes where the temperature induced elastic deformation occurs, biomedical engineering (problems involving thermal stress), emissions of the electromagnetic radiations from nuclear devices, development of a highly sensitive super conducting magnetometer, electrical power engineering, plasma physics etc. The interplay of the Maxwell electromagnetic field with the motion of deformable solids is largely being undertaken by many investigators owing to the possibility of its application to geophysical problems and certain topics in optics and acoustics. Moreover, the earth is subject to its own magnetic field and the material of the earth may be electrically conducting. Thus, the magneto-elastic nature of the earth's material may affect the propagation of waves. Many authors have considered the propagation of electro-magneto-thermoelastic waves in an electrically and thermally conducting solid. During the second half of 20th century, great attention has been devoted to the study of electromagneto-thermoelastic coupled problems based on the generalized thermoelasticity. The magneto-thermoelastic disturbances generated by a thermal shock in an elastic half-space having a finite conductivity has been investigated by Puri [6]. Among the authors who considered the generalized magneto-thermoelastic equations are Nayfeh and Nemat-Nasser [7] who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. They have obtained the governing equations in the general case and the solution for some particular cases. Choudhuri [8] extended these results to rotating media. Ezzat [9] has studied the problem of generation of generalized magneto-thermoelastic waves by thermal shock in a perfectly conducting half-space. Ezzat et al. [10] have established the model of two dimensional equations of generalized magneto-thermoelasticity. In dealing with classical or generalized thermoelastic problems in most situations, the displacement potential function approach is used. However Bahar and Hetnarski [11–12] outlined several disadvantages of the potential function approach. These may be summarized in the fact that the boundary and initial conditions of the problem are not related directly to the potential function, as it has no physical meaning explicitly.

Secondly, more stringent assumptions must be made on the behaviour of potential functions than on the actual physical quantities. Last of all, it was found that many integral representations of physical quantities are convergent in the classical sense while their potential function representations only converge in the mean. To get rid of these difficulties, Bahar and Hetnarski [13] introduced the state space formulation in thermoelastic problems. This state space approach has been further

developed in Sherief [14] to include the effect of heat sources. He et al. [15] surveyed a two-dimensional thermal shock problem for a semi-infinite piezoelectric rod using state space approach. Youssef and El-bary [16] put forward an analysis for a generalized thermoelastic infinite layer problem under three theories using state space approach. State space formulation to the vibration of gold nano-beam in femtoseconds scale was done by Elsibai and Youssef [17].

The theory of heat conduction in a deformable body, formulated by Chen and Gurtin [18] and Chen et al. [19,20] depends on two different temperatures the conductive temperature and the thermo dynamical temperature. Chen et al. [21] have suggested that the difference between these two temperatures is proportional to heat supply. In absence of heat supply, these two temperatures are identical for time independent situation. However, for time dependent cases, particularly for problems related to wave propagation, the two temperatures are in general different, regardless of heat supply. The two temperature thermoelasticity theory has gained much attention of the researchers in the recent years. The existence, structural stability, convergence and spatial behaviour of two temperature thermoelasticity have been provided by Quintanilla [22]. Youssef [23] has developed a new model of generalized thermoelasticity that depends on two temperatures T and ϕ , where the difference between the two temperatures is proportional to heat supply $\dot{\phi}_{,ii}$ with a non-negative constant.

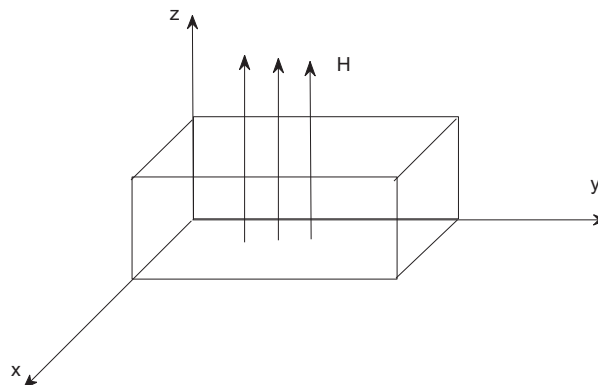
Roy Choudhuri and Debnath [24,25] and Othman [26,27]. Othman [28,29] studied the effect of rotation in a micropolar generalized thermoelastic and thermo-viscoelasticity half space under different theories. The propagation of plane harmonic waves in a rotating elastic medium without thermal field has been studied. It was shown there that the rotation causes the elastic medium to be dispersive and an isotropic. These problems are based on more realistic elastic model since earth; moon and other planets have angular velocity.

Owing to the mathematical difficulties encountered in two- and three dimensional multi-field coupled generalized heat conduction problems, the problems become too complicated to obtain an analytical solution. Instead of analytical methods, several authors have applied numerical methods such as finite difference method, finite element method and boundary value method etc. for solving such kind of problems. One can find several two-dimensional works based on the generalized thermoelasticity by using the normal mode analysis in the literatures Ezzat and Abd Elall [30], Othman and Lotfy [31–32], Lotfy and Othman [33], Lotfy [34] and Sarkar and Lahiri [35]. Using the normal mode analysis technique, we will get the solution in the Fourier transformed domain actually. To apply the normal mode analysis, we have to assume that all the relations are sufficiently smooth on the real axis such that the normal mode analysis of all these functions exist. The normal mode analysis [30–34] was used to obtain the exact expression for the temperature distribution, thermal stresses, and the displacement components.

The present article is concerned with a two-dimensional electro-magneto- thermoelastic coupled problem for a homogeneous, isotropic, thermally and electrically conducting half-space solid whose surface is subjected to a thermal shock with two temperature heat transfer. The normal mode analysis is used to obtain the exact analytical solutions of the problem. Numerical results for the temperature, displacements and thermal stresses distribution are presented graphically and discussed. A comparison is made with the results obtained in the presence and absence of two temperature parameter and the magnetic field effect.

2. Formulation of the problem and basic equations

Let us consider a perfectly conducting thermoelastic half-space $x \geq 0$ with two temperatures of a linear, homogeneous and isotropic medium whose state can be expressed in terms of the space variables x , y and the time variable t . The initial conditions are assumed to be homogeneous. The adjacent free space is assumed to be permeated by a uniform magnetic field intensity $\mathbf{H} = (0, 0, H_0)$ acts parallel to the bounding plane (table as the direction of the z -axis). acting parallel to the boundary $x = 0$. there are results of an induced magnetic field \mathbf{h} and an induced electric field \mathbf{E} , which satisfy the linearized equations of electro-magnetism and are valid for slowly moving media:



Geometry of the problem

$$\text{curl} \vec{h} = \vec{J} + \varepsilon_0 \vec{E}, \quad (1)$$

$$\text{curl} \vec{E} = -\mu_0 \dot{\vec{h}}, \quad (2)$$

$$\text{div} \vec{h} = 0, \quad (3)$$

$$\vec{E} = -\mu_0 (\dot{\vec{u}} \wedge \vec{H}), \quad (4)$$

where $\dot{\vec{u}}$ is the partial velocity of the medium, and the small effect of temperature gradient on \vec{J} is ignored. The dynamic displacement vector is actually measured from a steady state deformed position and the deformation is supposed to be small.

The components of the magnetic intensity vector in the medium are

$$H_x = 0, \quad H_y = 0, \quad H_z = H_0 + h(x, y, z). \quad (5)$$

The electric intensity vector is normal to both the magnetic intensity and the displacement vectors. Thus, it has the components

$$E_x = -\mu_0 H_0 \dot{v}, \quad E_y = \mu_0 H_0 \dot{u}, \quad E_z = 0. \quad (6)$$

The current density vector J be parallel to E , thus

$$J_x = \frac{\partial h}{\partial y} + \mu_0 H_0 \varepsilon_0 \ddot{v}, \quad J_y = -\frac{\partial h}{\partial x} - \mu_0 H_0 \varepsilon_0 \ddot{u}, \quad J_z = 0, \quad (7)$$

$$\vec{h} = -H_0(0, 0, e). \quad (8)$$

where e is the dilatation. If we restrict our analysis to plane strain parallel to xy -plane with displacement vector $\mathbf{u} = (u, v, 0)$.

The heat conduction equation takes the form [23]

$$K \varphi_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho C_E T + \gamma T_0 u_{i,j}) \quad (9)$$

The constitutive equation takes the form

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma \left(1 + \nu_0 \frac{\partial}{\partial t} \right) T \delta_{ij}. \quad (10)$$

The equation of motion without body force takes the form

$$\rho \ddot{u}_i = \sigma_{ij,j} + F_i, \quad (i, j = 1, 2, 3). \quad (11)$$

where $F_i = \mu_0 (J \times H)$.

The relation between the heat conduction and the dynamical heat takes the form

$$\varphi - T = a \varphi_{,ii}, \quad (12)$$

where $a > 0$ two-temperature parameter, Youssef [23].

Now, we will suppose elastic and homogenous half-space $x \geq 0$ which obey Eqs. (9)–(12) and initially quiescent where all the state functions are depend only on the dimension x, y and the time t .

The displacement components for one dimension medium have the form

$$u_x = u(x, y, t), \quad u_y = v(x, y, t) \text{ and } u_z = 0. \quad (13)$$

The strain component takes the form

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}). \quad (14)$$

The heat conduction equation takes the form

$$K \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right) = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \rho C_E T + \gamma T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right). \quad (15)$$

The constitutive equation takes the form

$$\sigma_{xx} = (2\mu + \lambda) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} - \gamma \left(1 + \nu_0 \frac{\partial}{\partial t} \right) T, \quad (16)$$

$$\sigma_{yy} = (2\mu + \lambda) \frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} - \gamma \left(1 + \nu_0 \frac{\partial}{\partial t} \right) T, \quad (17)$$

$$\sigma_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right). \quad (18)$$

Using the summation convection. From (16)–(18). For a two dimensional problem (xy -plane) all quantities depend only on space coordinates x, y and time t too. The field equations and constitutive relations in generalized linear thermoelasticity with the influence of magnetic field and without body forces and heat sources are

$$\rho \frac{\partial^2 u}{\partial t^2} = \mu \nabla^2 u + (\mu + \lambda) \frac{\partial e}{\partial x} - \gamma \left(1 + \nu_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x} - \mu_0 H_0^2 \frac{\partial e}{\partial x} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 u}{\partial t^2} \quad (19)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = \mu \nabla^2 v + (\mu + \lambda) \frac{\partial e}{\partial y} - \gamma \left(1 + \nu_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial y} - \mu_0 H_0^2 \frac{\partial e}{\partial y} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 v}{\partial t^2}. \quad (20)$$

The relation between the heat conduction and dynamical heat takes the form

$$\varphi - T = a \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right) \quad (21)$$

For simplicity, we will use the following non-dimensional variables

$$\begin{aligned} (x', y', u', v') &= c_0 \eta (x, y, u, v), (t', \tau'_0, v'_0) = c_0^2 \eta (t, \tau_0, v_0), (\theta', \varphi') = \frac{(T, \varphi) - T_0}{T_0}, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{2\mu + \lambda}, \quad g' = \frac{g}{C_0 \eta}, \quad \bar{h} \\ &= \frac{h}{H_0}, \end{aligned} \quad (22)$$

where $\eta = \frac{\rho C_E}{K}$, $C_2^2 = \frac{\mu}{\rho}$ and $C_0^2 = \frac{2\mu + \lambda}{\rho}$. Hence, we have (dropping the dashed for convenience)

$$\nabla^2 \varphi - \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial \theta}{\partial t} - \varepsilon \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial e}{\partial t} = 0 \quad (23)$$

$$\varphi - \theta = \beta \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right), \quad (24)$$

And the equations of motion take the form

$$\alpha \frac{\partial^2 u}{\partial t^2} = a_1^* \nabla^2 u + a_2 \frac{\partial e}{\partial x} - a_0 \left(1 + \nu_0 \frac{\partial}{\partial t} \right) \frac{\partial \theta}{\partial x} \quad (25)$$

$$\alpha \frac{\partial^2 v}{\partial t^2} = a_1^* \nabla^2 v + a_2 \frac{\partial e}{\partial y} - a_0 \left(1 + \nu_0 \frac{\partial}{\partial t} \right) \frac{\partial \theta}{\partial y} \quad (26)$$

where $\varepsilon = \frac{\gamma}{\rho C_E}$ and $\beta = a \eta^2 c_0^2$, $a_1^* = \frac{\mu}{\rho C_0^2}$, $a_2^* = \frac{\mu + \lambda}{\rho C_0^2}$, $a_0 = \frac{\gamma T_0}{\rho C_0^2}$, $\alpha = 1 + \frac{\mu_0^2 H_0^2 \varepsilon_0}{\rho}$, $R_H = \frac{\mu_0 H_0^2}{\rho C_0^2}$, $a_2 = a_2^* - R_H$.

Assuming the scalar potential functions $\Pi(x, y, t)$ and $\psi(x, y, t)$ defined by the relations in the non-dimensional form:

$$u = \frac{\partial \Pi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \Pi}{\partial y} - \frac{\partial \psi}{\partial x}. \quad (27)$$

By using (27) and (22) in Eqs. (25) and (26), we obtain.

$$\left[\nabla^2 - a_3 \frac{\partial^2}{\partial t^2} \right] \Pi - a_0 \left(1 + \nu_0 \frac{\partial}{\partial t} \right) \theta = 0 \quad (28)$$

$$\left(\nabla^2 - a_4 \frac{\partial^2}{\partial t^2} \right) \psi = 0 \quad (29)$$

where $a_3 = \frac{\alpha}{a_1^* + a_2}$, $a_4 = a_1 \alpha$, $a_1 = \frac{1}{a_1^*}$, also Eq. (15) takes the form

$$\nabla^2 \varphi - \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial \theta}{\partial t} - \varepsilon \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial \Pi}{\partial t} = 0 \quad (23^*)$$

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$[\Pi, \psi, \varphi, \theta, \sigma_{ij}](x, y, t) = [u^*(x), \varphi^*(x), \theta^*(x), \sigma_{ij}^*(x)] \exp(\omega t + iby), \quad (30)$$

where ω is the (complex) time constant. $i = \sqrt{-1}$, b be a wave number and $u^*(x)$, $\varphi^*(x)$, $\theta^*(x)$ and $\sigma_{ij}^*(x)$ are the amplitude of the field quantities. Using Eqs. (30), (28), (29), (23)* and (24) takes the form

$$[D^2 - A_1]\Pi^* - A_2\theta^* = 0, \quad (31)$$

$$(D^2 - A_4)\psi^* = 0, \quad (32)$$

$$[D^2 - A_3]\varphi^* = -\beta^*\theta^*, \quad (33)$$

$$(D^2 - b^2)\varphi - A\theta^* - B\Pi^* = 0, \quad (34)$$

where $A = \omega(1 + \omega\tau_0)$, $B = \varepsilon A$, $A_1 = b^2 + a_3\omega^2$, $A_2 = a_0(1 + \nu_0\omega)$, $A_3 = (\beta b^2 + 1)/\beta$, $\beta^* = \frac{1}{\beta}$, $A_4 = b^2 + a_4\omega^2$ and $D = \frac{d}{dx}$. Eliminating $\theta^*(x)$, $\Pi^*(x)$, and $\varphi^*(x)$ between Eqs. (31), (33), and (34), we obtain the partial differential equation satisfied by $\theta^*(x)$

$$[D^4 - ED^2 + F]\theta^*(x) = 0. \quad (35)$$

where $A_6 = \frac{(\beta^* b^2 + A_3 A)}{\beta^* + A}$, $A_7 = -\frac{B}{\beta^* + A}$.

Since

$$E = \frac{(A_1 + b^2)(\beta^* + A) - BA_2}{\beta^* + A}, \quad (36)$$

$$F = \frac{b^2 A_1 (\beta^* + A_1) - BA_2 A_3}{\beta^* + A}, \quad (37)$$

In a similar manner, we get

$$[D^4 - ED^2 + F](\Pi^*, \varphi^*)(x) = 0. \quad (38)$$

The above equation can be factorized

$$(D^2 - k_1^2)(D^2 - k_2^2)\theta^*(x) = 0, \quad (39)$$

where, $k_n^2 (n = 1, 2)$ are the roots of the following characteristic equation

$$k^4 - Ek^2 + F = 0. \quad (40)$$

The solution of Eq. (39) which is bounded as $x \rightarrow \infty$, is given by

$$\theta^*(x) = \sum_{n=1}^2 M_n(b, \beta, \omega) \exp(-k_n x). \quad (41)$$

Similarly

$$\varphi^*(x) = \sum_{n=1}^2 M'_n(b, \omega) \exp(-k_n x) \quad (42)$$

$$\Pi^*(x) = \sum_{n=1}^2 M''_n(b, \omega) \exp(-k_n x) \quad (43)$$

The solution of Eq. (35) can be written as

$$\psi^*(x) = M_3 e^{-mx} \quad (44)$$

since,

$$u^*(x) = D\Pi^* + ib\psi^*, \quad (45)$$

$$v^*(x) = ib\Pi^* - D\psi^*, \quad (46)$$

$$e^*(x) = Du^* + ibv^*. \quad (47)$$

Using Eqs. (45) and (46), in order to obtain the amplitude of the displacement components u and v , which are bounded as $x \rightarrow \infty$, then Eqs. (45) and (46) become

$$u^*(x) = -\sum_{n=1}^2 M_n''(b, \beta, \omega) k_n e^{-k_n x} + i b M_3 e^{-m x}, \quad (48)$$

$$v^*(x) = \sum_{n=1}^2 i b M_n''(b, \beta, \omega) e^{-k_n x} - m M_3 e^{-m x}. \quad (49)$$

where M_n, M_n' and M_n'' are some parameters depending on β, a and ω .

Substituting from Eqs. (41)–(43) into Eqs. (32)–(34), we have

$$M_n'(b, \omega) = H_{1n} M_n(b, \omega), \quad n = 1, 2. \quad (50)$$

$$M_n''(b, \omega) = H_{2n} M_n(b, \omega), \quad n = 1, 2. \quad (51)$$

where

$$H_{1n} = \left(\frac{\beta^*}{A_3 - k_n^2} \right), \quad n = 1, 2. \quad (52)$$

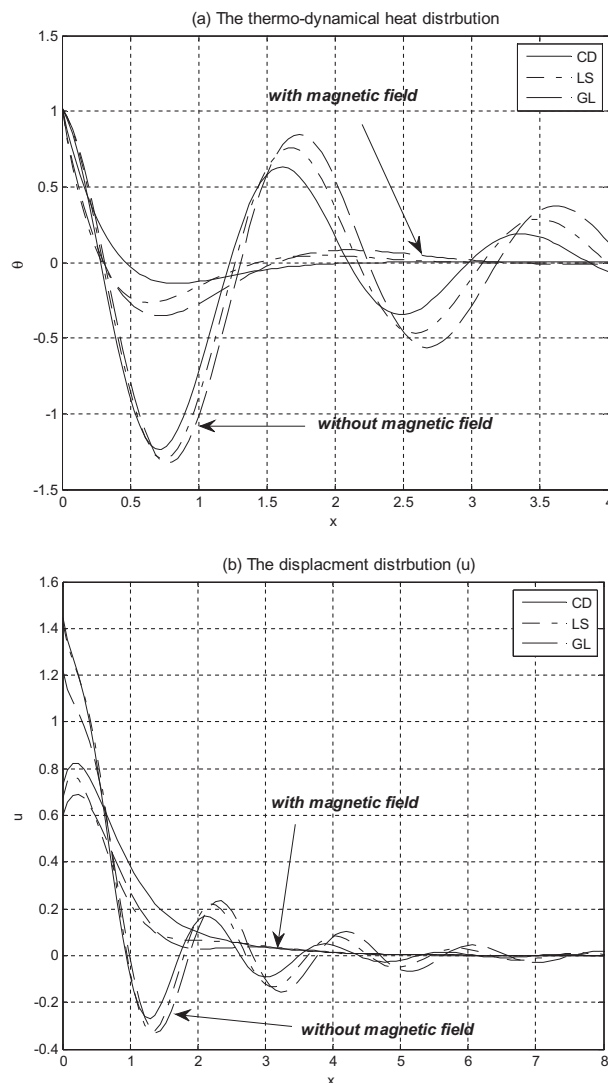


Fig. 1. (a,b) The thermal temperature θ and the displacement distribution with and without magnetic field when $\beta = 0.1$ and $t = 0.1$ under three theories. (c,d) The stresses distribution σ_{yy} and σ_{xx} distribution with and without magnetic field when $\beta = 0.1$ and $t = 0.1$ under three theories. (e,f) The stresses distribution σ_{xy} and the thermal temperature ϕ distribution with and without magnetic field when $\beta = 0.1$ and $t = 0.1$ under three theories.

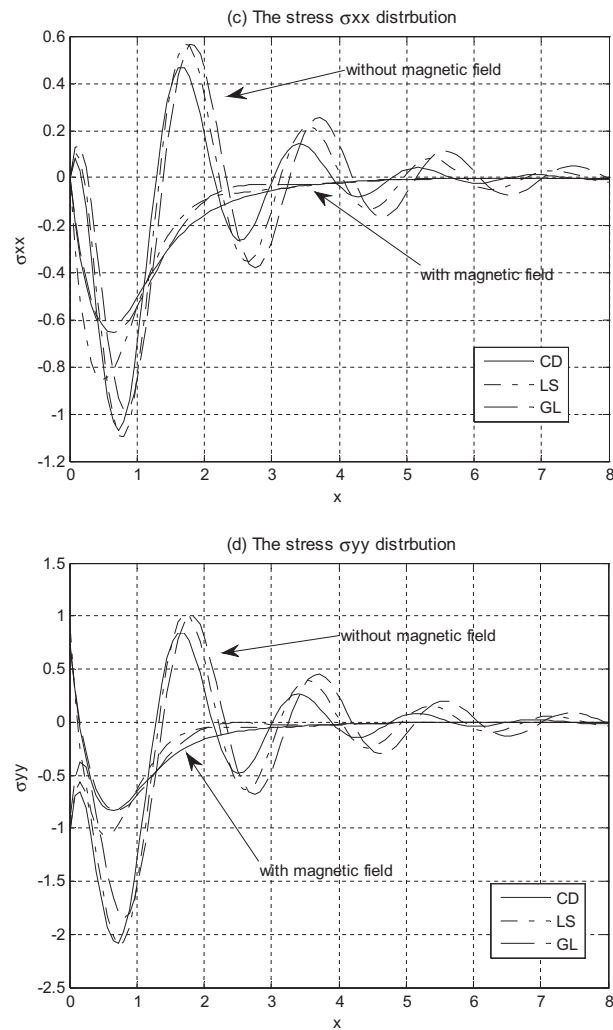


Fig. 1 (continued)

$$H_{2n} = \frac{A_2}{(k_n^2 - A_1)}, \quad n = 1, 2. \quad (53)$$

Thus, we have

$$\varphi^*(x) = \sum_{n=1}^2 H_{1n} M_n(b, \beta^*, \omega) \exp(-k_n x) \quad (54)$$

$$\Pi^*(x) = \sum_{n=1}^2 H_{2n} M_n(b, \beta^*, \omega) \exp(-k_n x) \quad (55)$$

Substitution of Eqs. (22), (30), (48), and (49) into Eqs. (16)–(18), we get

$$\sigma_{xx}^* = \sum_{n=1}^2 h_n M_n(b, \omega) \exp(-k_n x) - q_1 M_3 \exp(-mx), \quad (56)$$

$$\sigma_{yy}^* = \sum_{n=1}^2 h'_n M_n(b, \omega) \exp(-k_n x) + q_1 M_3 \exp(-mx), \quad (57)$$

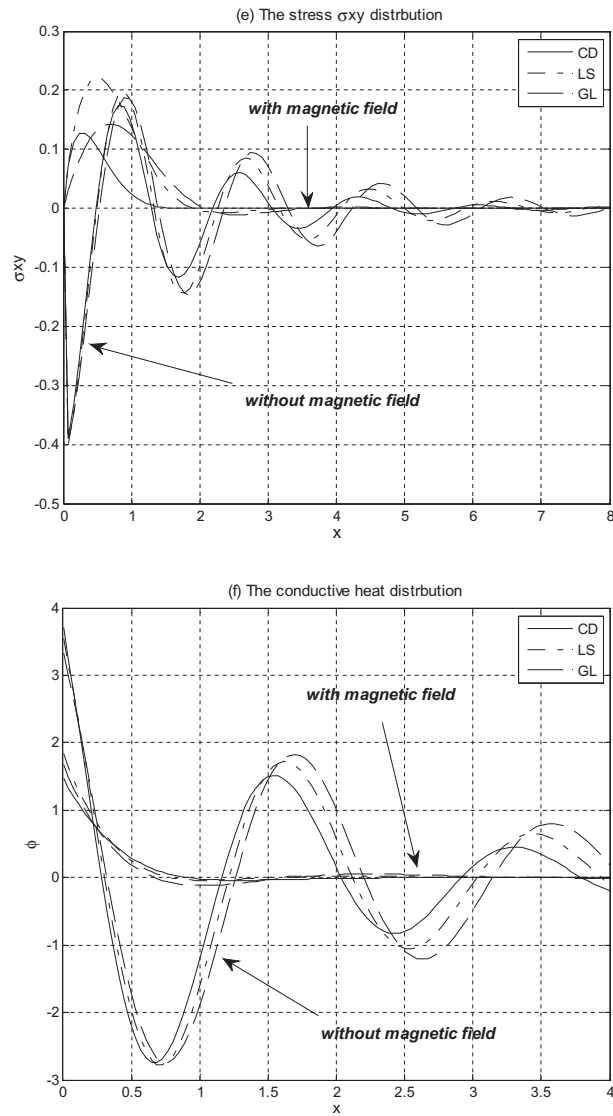


Fig. 1 (continued)

$$\sigma_{xy}^* = \sum_{n=1}^2 h_n'' M_n(b, \omega) \exp(-k_n x) + q_2 M_3 \exp(-mx), \quad (58)$$

$$u^*(x) = \sum_{n=1}^3 -k_n H_{2n} M_n(b, \omega) e^{-k_n x} + i b M_3 e^{-mx}, \quad (59)$$

$$v^*(x) = \sum_{n=1}^2 i b H_{2n} M_n(b, \omega) e^{-k_n x} + m M_3 e^{-mx}, \quad (60)$$

where

$$h_n = - \left(-H_{2n} k_n^2 + \frac{b^2 \lambda H_{2n}}{2\mu + \lambda} + \frac{\gamma T_0}{2\mu + \lambda} \right), \quad (61)$$

$$h'_n = - \left(b^2 H_{2n} - \frac{\lambda H_{2n} k_n^2}{2\mu + \lambda} + \frac{\gamma T_0}{2\mu + \lambda} \right), \quad (62)$$

$$h''_n = \frac{-2ib\mu H_{2n}}{2\mu + \lambda} k_n, \quad (63)$$

$$q_1 = ibm \left(1 - \frac{\lambda}{2\mu + \lambda} \right), \quad (64)$$

$$q_2 = \frac{-\mu}{2\mu + \lambda} (m^2 + b^2). \quad (65)$$

The normal mode analysis is, in fact, to look for the solution in Fourier transformed domain. Assuming that all the field quantities are sufficiently smooth on the real line such that normal mode analysis of these functions exists.

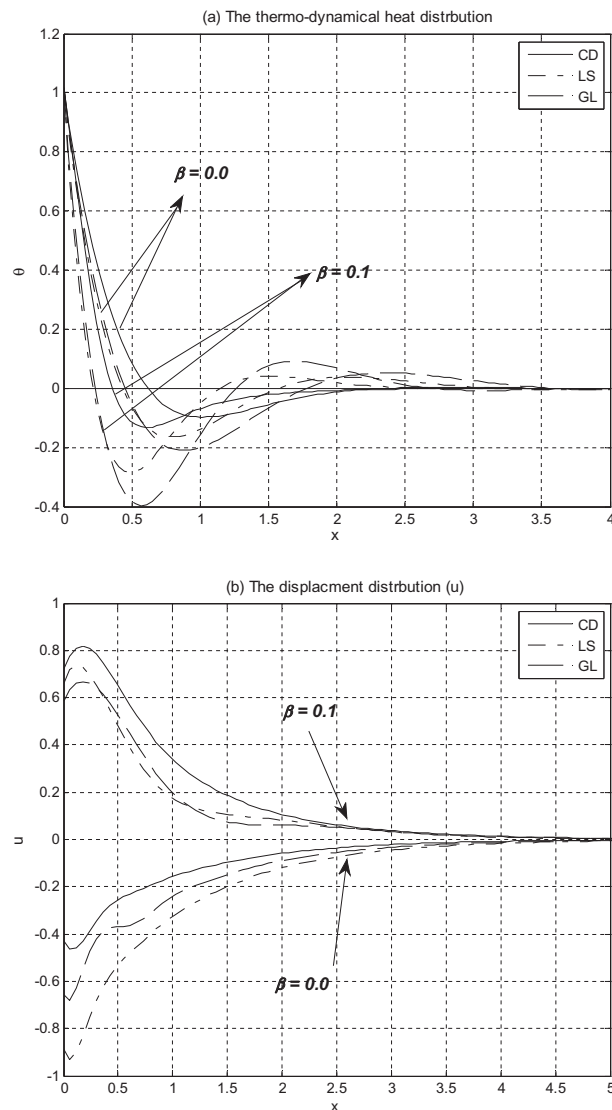


Fig. 2. (a,b) The thermal temperature θ and the displacement distribution with different values of two-temperature parameter at the constant of magnetic field and $t = 0.1$ under three theories. (c,d) The stresses distribution σ_{yy} and σ_{xx} with different values of two-temperature parameter at the constant of magnetic field and $t = 0.1$ under three theories. (e,f) The stresses distribution σ_{xy} and the thermal temperature ϕ distribution with different values of two-temperature parameter at the constant of magnetic field and $t = 0.1$ under three theories.

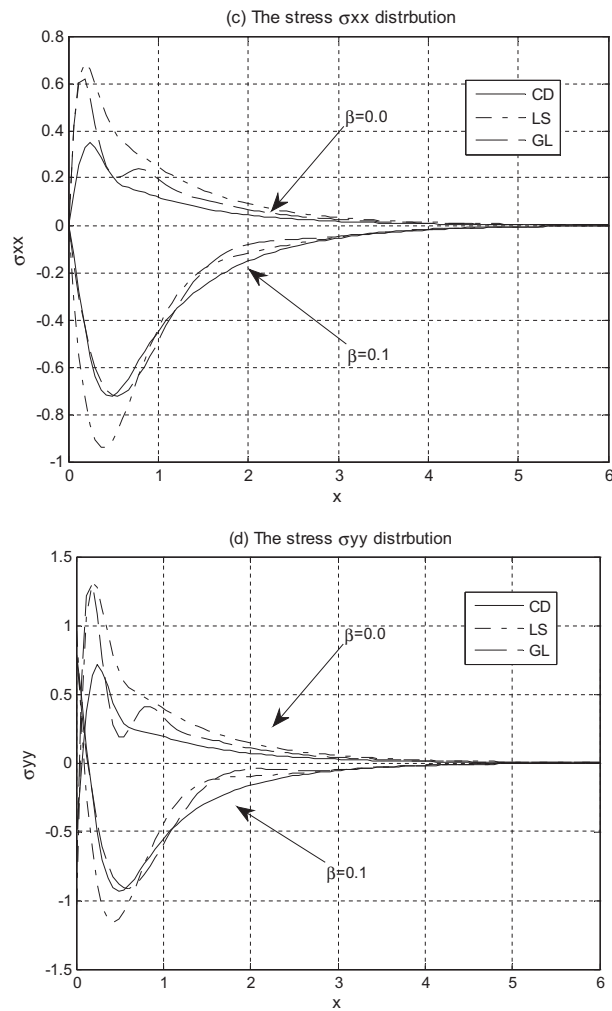


Fig. 2 (continued)

3. Application

In this section we determine the parameters M_n ($n = 1, 2, 3$). In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. The constants M_1, M_2, M_3 have to chosen such that the boundary conditions on the surface at $x = 0$ take the form

- (1) Thermal boundary conditions that the surface of the half-space subjected to thermal shock

$$\theta(0, y, t) = f(0, y, t). \quad (66)$$

- (2) Mechanical boundary condition that surface of the half-space is traction free

$$\sigma_{xx}(0, y, t) = 0. \quad (67)$$

- (3) Mechanical boundary condition that surface of the half-space is traction free

$$\sigma_{xy}(0, y, t) = 0. \quad (68)$$

Substituting the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{n=1}^3 H_{1n} M_n(b, \beta^*, \omega) = f^*(y, t), \quad (69)$$

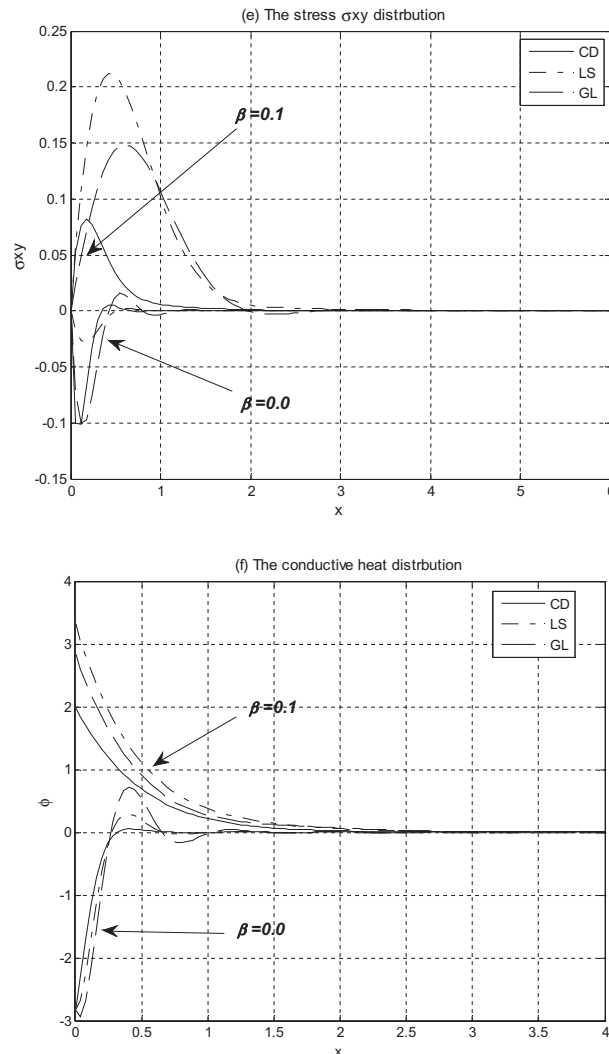


Fig. 2 (continued)

$$\sum_{n=1}^3 h_n M_n(b, \beta, \omega) = 0, \quad (70)$$

$$\sum_{n=1}^3 h_n'' M_n(b, \beta, \omega) = 0, \quad (71)$$

where f is the magnitudes of thermal source. Invoking the boundary conditions (69)–(71) at the surface $x = 0$ of the plate, we obtain a system of three equations. After applying the inverse of matrix method (by mat lap program), we can obtain the values of the three constants $M_j, j = 1, 2, 3$. Hence, the expressions for the displacements, temperature distribution and other physical quantity of the medium can be obtained.

4. Numerical results

In order to analyze the above problem numerically, we now consider a numerical example for which computational results are given. The results depict the variation of temperature, displacement and stress fields in the context of two theories. To study the effect of rotation and two temperature on wave propagation. Since we have $\omega = \omega_0 + i\xi$ where the imaginary unit is $i = \sqrt{-1}$, $e^{\omega t} = e^{\omega_0 t}(\cos \xi t + i \sin \xi t)$ and for small value of time, we can take $\omega = \omega_0$ (real). The copper material was chosen for the purpose of numerical example. The numerical constants (in SI unit) of the problem were taken as:

$$\lambda = 7.59 \times 10^9 \text{ N/m}^2, \quad \mu = 3.86 \times 10^{10} \text{ kg/ms}^2, \quad \rho = 8954 \text{ kg/m}^3, \quad \tau_0 = 0.02 \text{ s}$$

$$\alpha = -1.28 \times 10^9 \text{ N/m}^2, \quad \beta = 0.32 \times 10^9 \text{ N/m}^2, \quad \eta = 8886.73 \text{ m/s}^2, \quad \varepsilon = 0.0168$$

$$\alpha_t = 1.78 \times 10^{-5} \text{ K}^{-1}, \quad k = 386 \text{ Wm}^{-1} \text{ K}^{-1}, \quad b = 1, \quad C_E = 383.1 \text{ J/(kgk)}$$

$$T_0 = 293 \text{ K}, \quad f^* = 1, \quad \omega = \omega_0 + i\xi, \quad \omega_0 = 2, \quad \xi = 1.$$

The computations were carried out for a value of time $t = 0.1$. The numerical technique, outlined above, was used for the distribution of the real part of the thermal temperature θ and ϕ , the displacement u, v , strain and the stress ($\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$) distribution for the problem. The field quantities, temperature, displacement components and stress components depend not only on space x and time t but also on the thermal relaxation time τ_0 . Here all the variables are taken in non dimensional forms.

The first group Fig. 1(a)–(f), the graph shows the three curves predicted by different theories of thermoelasticity under the effect of magnetic field and constant of two temperature parameter ($\beta = 0.1$). In these figures, the solid lines represent the solution in the Coupled (CD) theory, the dashed-dot lines represent the solution in the Lord and Shulman (LS) theory with one relaxation time and the dashed lines represent the solution in the generalized Green–Lindsay (GL) theory with two relaxation times. We notice that the results for the temperature, the displacement and stresses distribution when the relaxation time is including in the heat equation are distinctly different from those when the relaxation time is not mentioned in heat equation, because the thermal waves in the Fourier's theory of heat equation travel with an infinite speed of propagation as

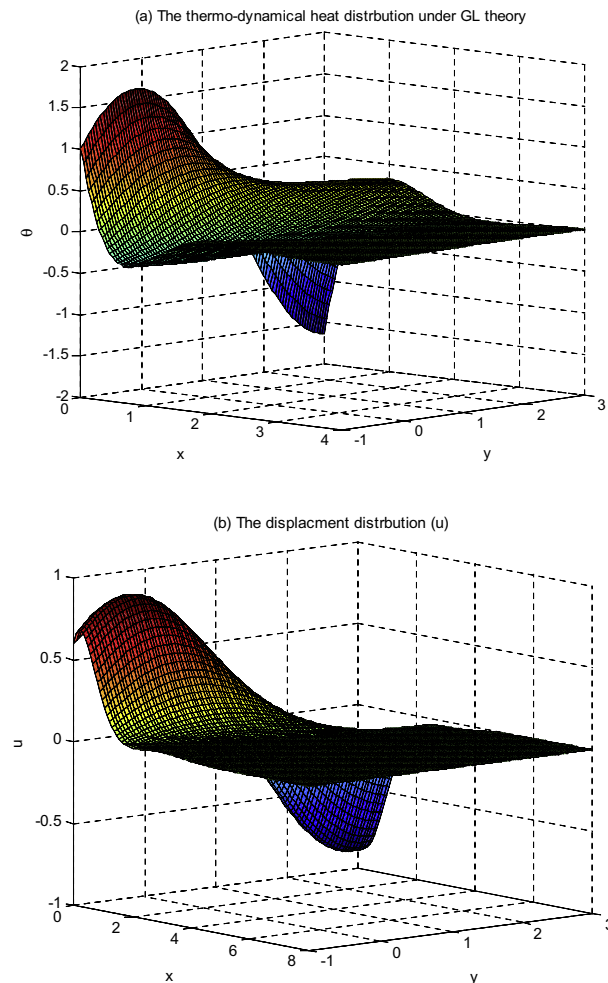


Fig. 3. (a,b) The 3D thermal temperature θ and the displacement u distribution with magnetic field when $\beta = 0.1$ and $t = 0.1$ under GL theory. (c,d) The 3D stresses distribution σ_{yy} and σ_{xy} distribution with magnetic field when $\beta = 0.1$ and $t = 0.1$ under GL theory.

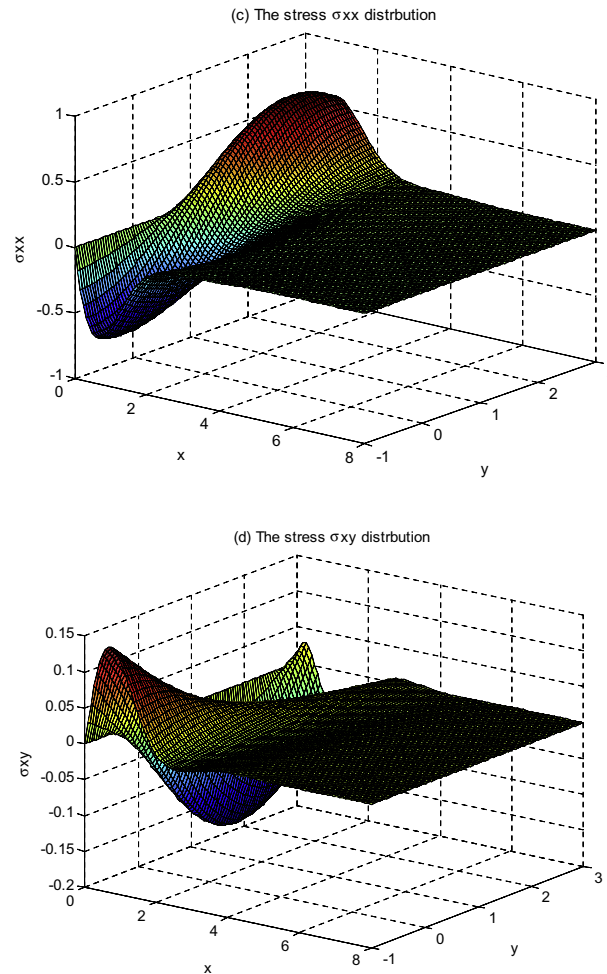


Fig. 3 (continued)

opposed to finite speed in the non-Fourier case. By comparing figures of solutions obtained under the three thermoelastic theories, important phenomena are observed:

- (1) The curves in the context of the LS, GL and CD theories decrease exponentially with increasing distance x , indicating that the thermoelastic waves are unattenuated and nondispersive, where purely thermoelastic waves undergo both attenuation and dispersion.
- (2) The values of solutions for GL theory are large in comparison with those for LS theory. In fact, for GL theory, the relaxation time ν_0 is large ($\nu_0 > \tau_0 > 0$), so the time available for the exchange of thermal energy with the domain is large and the values of solutions are thus higher.

It should be noted (Fig. 1(a)) that in this problem, it is clear from the graph that θ decreases smoothly to minimum value at the beginning, where it experiences smooth increases (with maximum positive gradient) and then oscillate uniformly (without magnetic field). The variations of thermo-dynamical heat distribution θ (with magnetic field) are not similar in nature for the influence of magnetic field in the medium with difference in magnitude. In other words, the temperature lines without magnetic field have the highest gradient when compared with that of with magnetic field in all range. In addition, all lines begin to coincide when the horizontal distance x increases to reach the reference temperature of the solid. These results obey physical reality for the behavior of copper as a polycrystalline solid. Fig. 1(b), the values of the horizontal displacement u for the thermoelastic medium under the influence of magnetic field, the magnitude of the maximum displacement peak strongly depends on the magnetic field. It is also clear that the rate of change of u decreases with increasing the magnetic field. On the other hand, these variations are oscillatory in nature. Fig. 1(c), shows that the normal stress component σ_{xx} , take the different behavior. In other words, the σ_{xx} component line for present the magnetic field (take a exponential function) has the lowest gradient when compared with that of absence (take a wave function) and satisfies the boundary

conditions. That the variations of σ_{xx} for a thermoelastic medium without magnetic field are more oscillatory in nature as compared with those variations obtained with the effect of Fig. 1(d) the horizontal stresses σ_{yy} . Graph lines, under the influence of magnetic field and three theories, the values of force stress σ_{yy} first increases sharply in the first range and then sharp decreases and hence oscillate uniformly when the magnetic field is absent. Also the values force stress σ_{yy} for a thermoelastic medium with magnetic field satisfies the boundary condition. In other words, the σ_{yy} component line for magnetic has the lowest gradient when compared with that of absent the magnetic field. In addition, all lines begin to coincide when the horizontal distance x is increases to reach zero after their relaxations at infinity. Variation of magnetic field has a serious effect on both magnitudes of mechanical stresses. These trends obey elastic and thermoelastic properties of the solid under investigation. Fig. 1(e), shows that the stress component σ_{xy} satisfy the boundary condition, it sharp decreases in the start and start increases (maximum) in the context of the magnetic field is absent but when present take the different behavior. The lines for Fig. 1(f) displays the conductive temperature in which we observe the significant difference in the conductive temperature is noticed for the value of the magnetic field. The conductive temperature begins from the positive values and then decreases to arrive the minimum amplitudes in two cases of variation magnetic field, also move in the wave propagation when the magnetic field is absent and exponential function when presented, beyond it falls again to try to retain zero at infinity.

The second group Fig. 2(a)–(f) show the comparison between the thermal temperature θ and ϕ , displacement component (u) and the stress (σ_{xx} , σ_{yy} , σ_{xy}) distribution, the case of different two values of two temperature parameter. For the value of y , namely $y = -1$, $t = 0.1$ and the magnetic field is present fewer than three theories, were substituted in performing the computation. Fig. 2(a) exhibit the space variation of temperature distribution in which we observe the following: Significant

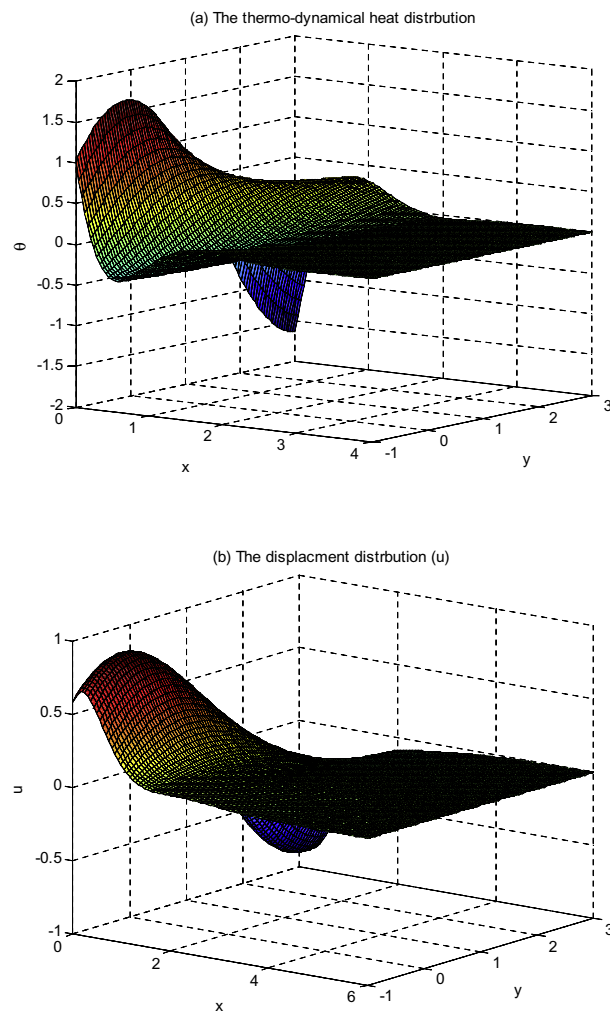


Fig. 4. (a,b) The 3D thermal temperature θ and the displacement distribution with two-temperature parameter at the constant of magnetic field and $t = 0.1$ under GL theory. (c,d) The 3D stresses distribution σ_{xx} and the thermal temperature ϕ distribution with two-temperature parameter at the constant of magnetic field and $t = 0.1$ under GL theory.

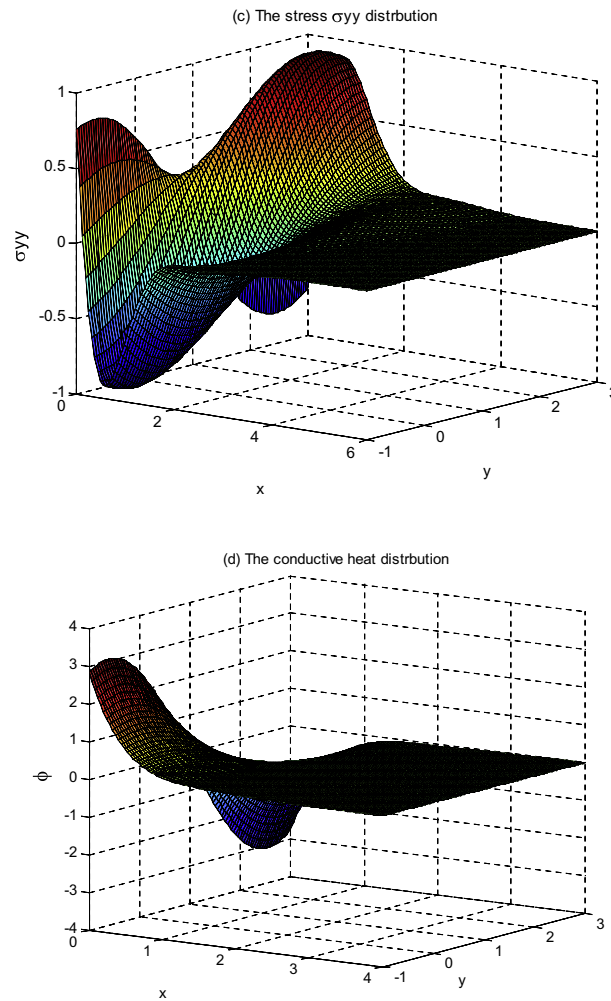


Fig. 4 (continued)

difference in the thermo-dynamical temperature is noticed for different value of the nondimensional two-temperature parameter. It should be noted (fig. 2(a)). It is clear from the graph that θ has decreases to arrive the minimum value at the beginning in two cases $\beta = 0.0$ (one temperature) and $\beta = 0.1$ (two temperature parameter). The value of temperature quantity converges to zero with increasing the distance x and satisfies the boundary conditions at $x = 0$. Also from this figure we can see, when $\beta = 0.0$ moves in the exponential function and also when $\beta = 0.1$. Fig. 2(b) the horizontal displacement u , we see that the displacement component u always starts from the negative value when $\beta = 0.0$ and positive value when $\beta = 0.1$ and terminates at the zero value, begins with increase smoothly to reach its maximum magnitude under two temperature parameter. Beyond it u falls again to try to retain zero at infinity, beyond reaching zero at the infinity (state of particles equilibrium when $\beta = 0.1$). The displacements u show the different behaviors at different values of β . The stress component, σ_{xx} reach coincidence with zero value (fig. 2(c)) of two cases which show the different behaviours when $\beta = 0.1$ and $\beta = 0.0$, then oscillate with decreasing behaviors. We can say, significant difference in the stress component σ_{xx} is noticed for different values of the non-dimensional two-temperature parameter. In addition, all lines begin to coincide when the horizontal distance x increases to reach zero at infinity. The stress component distribution is continuous, smooth and moves in the exponential function. These trends obey elastic and thermoelastic properties of the solid. The stress component, σ_{yy} start from the negative value (fig. 2(d)), reach the minimum value in the beginning and sharp increases then decreases smoothly and converges to zero with increasing the distance x (when $\beta = 0.0$). While σ_{yy} start from the positive value, it decreases sharply in the start and arrive to minimum in the context of the two temperature values (when $\beta = 0.1$). The two cases are different the exponential function. Fig. 2(e) the stress components σ_{xy} satisfies the boundary condition and start from zero. It smooth increases in the start to arrived the maximum and then start smooth decreases to minimum when $\beta = 0.1$ but decreases in the start to arrived the minimum and then start smooth increases to maximum when $\beta = 0.0$. Fig. 2(f) displays the conductive temperature in which we observe the Significant difference in the conductive

temperature is noticed for the value of the non-dimensional two temperature parameter β where the case of $\beta = 0.1$ and $\beta = 0.0$ indicates the new case (two-temperature). The conductive temperature begins from the positive value and then decreases to arrive the minimum amplitudes when $\beta = 0.1$, also move in the exponential propagation when $\beta = 0.1$, beyond it falls again to try to retain zero at infinity.

Finally, Figs. 3 and 4(a)–(d) plot in 3D the variations of the temperature θ , ϕ , the displacement component u , the stresses σ_{xx} , σ_{yy} and σ_{xy} distribution with the axes (x, y) . It is concluded that all the values decreases with an increasing of smaller values of y -axis and with the larger values increase. It is also clear that the temperature θ , ϕ , displacement and normal stress σ_{yy} component have been start from positive values, decrease with the smaller values of x -axis and return to increase to tend zero as x tends to infinity but the shear stress component σ_{xx} , start from zero and satisfy the boundary conditions, increases with the smaller values of x -axis and return twice to decreasing and increasing periodically to tends to zero as x tends to infinity.

5. Conclusions

The curves of the physical quantities with (CD) theory in most of figures are lower in comparison with those under (LS) theory and (GL) theory, due to the relaxation times. Analytical solutions based upon normal mode analysis for thermoelastic problem in solids have been developed and utilized. The theory of two-temperature generalize thermoelasticity describes the behavior of the particles of the elastic body more real than the theory of one-temperature generalized thermoelasticity. The context of the theory of two-temperature. The value of all the physical quantities converges to zero with an increase in distance x and All functions are continuous. Deformation of a body depends on the nature of forced applied as well as the type of boundary conditions. It is clear from all the figures that all the distributions considered have a non-zero value at the beginning except in a bounded region of the half-space. Outside of this region, the values vanish identically and this means that the region has not felt thermal disturbance yet. From the temperature distributions, we have found a wave and exponential type heat propagation with finite speeds in the medium. The heat wave front moves forward with a finite speed in the medium with the passage of time which proves that the generalized thermoelasticity theory with two temperature heat transfer is much closed to the behavior to the elastic materials. This is not the case for the CD theory where an infinite speed of thermal propagation can be found and hence all the considered physical quantities have a non-zero (possibly very small) value for any point in the medium. This indicates that the generalized Fourier's heat conduction mechanism is completely different from the classical Fourier's law. The effect of magnetic field in the medium plays significant role in the study of deformation of a body. There is much difference in the variations of quantities for the GL, LS and CD theories of thermoelasticity.

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